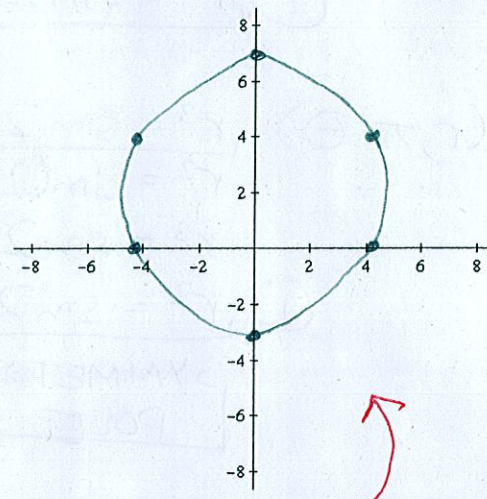


Consider the conic with polar equation $r = \frac{21}{5 - 2 \sin \theta}$.

- [a] Find the eccentricity of the conic. What is the shape of the graph?

$$r = \frac{\frac{21}{5}}{1 - \frac{2}{5} \sin \theta} \rightarrow e = \frac{2}{5} \rightarrow \text{ELLIPSE}$$

① ① ①



- [b] Find the equation of the directrix.

$$ep = \frac{21}{5}$$

$$\frac{2}{5}p = \frac{21}{5}$$

$$p = \frac{21}{2}$$

①

$$y = -\frac{21}{2}$$

① ① ①

- [c] Find the rectangular co-ordinates of all x - and y - intercepts of the conic.

θ	r	(x, y)
0	$\frac{21}{5}$	$(\frac{21}{5}, 0)$
$\frac{\pi}{2}$	7	$(0, 7)$
π	$\frac{21}{5}$	$(-\frac{21}{5}, 0)$
$\frac{3\pi}{2}$	3	$(0, -3)$

②

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ME
(REMEMBER
TO ADD
THE POINTS IN)
TO YOUR SCORE

- [d] Find the rectangular co-ordinates of the endpoints of all latera recta of the conic.

CENTER $(0, \frac{7-3}{2}) = (0, 2)$

FOCUS $(0, 2 \times 2) = (0, 4)$

ENDPOINTS OF L.R. = $(\pm \frac{21}{5}, 0), (\pm \frac{21}{5}, 4)$

① ②

- [e] Sketch the graph on the rectangular graph paper on the upper right.

Using only the standard tests and shortcuts shown in lecture, determine if the graph of $r^3 = \sin 2\theta$ is symmetric over the pole, the polar axis and/or $\theta = \frac{\pi}{2}$. Summarize your conclusions in the table on the right.

SCORE: ____ / 6 PTS

NOTE: You should run as few tests as needed to prove your conclusions are correct.

$$(-r, -\theta): (-r)^3 = \sin 2(-\theta) \quad (1)$$

$$-r^3 = \sin(-2\theta)$$

$$-r^3 = -\sin 2\theta$$

$$(1) \quad r^3 = \sin 2\theta \quad \text{SYMMETRIC OVER } \theta = \frac{\pi}{2} \quad (2)$$

Type of symmetry	Conclusion
Over the pole	SYMMETRIC
Over the polar axis	SYMMETRIC
Over $\theta = \frac{\pi}{2}$	SYMMETRIC

$$(r, \pi + \theta): r^3 = \sin 2(\pi + \theta) \quad (1)$$

$$r^3 = \sin(2\pi + 2\theta)$$

$$r^3 = \sin 2\pi \cos 2\theta + \cos 2\pi \sin 2\theta$$

$$(1) \quad r^3 = \sin 2\theta$$

SYMMETRIC OVER POLE (1) (2)

2 TYPES OF SYMMETRY, SO AUTOMATICALLY SYMMETRIC OVER POLAR AXIS (1)

Find all values of $\theta \in [0, 2\pi]$ for which the graph of $r = \cos 2\theta$ passes through the pole.

SCORE: ____ / 4 PTS

$$\cos 2\theta = 0 \quad (1)$$

$$(2) \quad 2\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$$

$$(1) \quad \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

$$0 \leq \theta \leq 2\pi$$

$$0 \leq 2\theta \leq 4\pi$$

GRADE AGAINST ONE SOLUTION ONLY

$$(1) \quad \cos 2\theta = 0$$

$$(2) \quad 2\cos^2\theta - 1 = 0$$

$$(1) \quad \cos\theta = \pm \frac{1}{\sqrt{2}} = \pm \frac{\sqrt{2}}{2}$$

$$(2) \quad \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

Consider the graph of the polar equation $r = 2 - 3\cos\theta$. $|\frac{2}{-3}| < 1$

SCORE: ____ / 6 PTS

[a] The shape of the graph is a/an LIMACON WITH LOOP (1)

[b] Find the rectangular coordinates of all x - and y -intercepts.

$$\theta \quad r \quad (x, y)$$

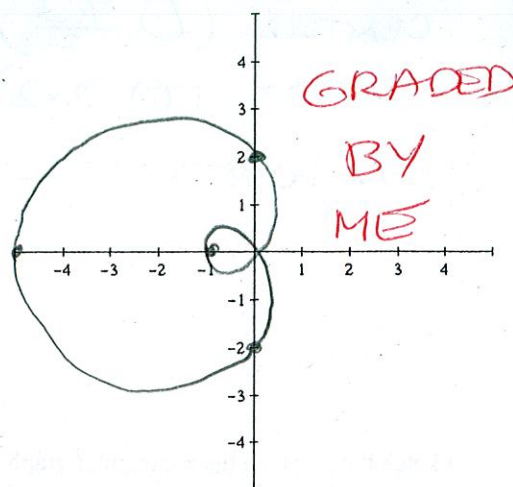
$$0 \quad -1 \quad (-1, 0)$$

$$\frac{\pi}{2} \quad 2 \quad (0, 2)$$

$$\pi \quad 5 \quad (-5, 0)$$

$$\frac{3\pi}{2} \quad 2 \quad (0, -2)$$

(2)



[c] Sketch the graph on the rectangular graph paper on the right.